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INTERNATIONAL CONFERENCE ON SEMIGROUPS AND AUTOMATA CSA 2016

Celebrating the 60th birthday
of Jorge Almeida and Gracinda Gomes

INVITED SPEAKERS

João Araújo (Univ. Aberta)
Karl Auinger (Univ. Vienna)
José Carlos Costa (Univ. Minho)
Volker Diekert (Univ. Stuttgart)
Ruy Exel (Univ. Federal Santa Catarina)
Victoria Gould (Univ. York)
Robert Gray (Univ. East Anglia)
Mark Kambites (Univ. Manchester)
Ganna Kudryavtseva (Univ. Ljubljana)
Mark Lawson (Heriot-Watt Univ.)
Markus Lohrey (Univ. Siegen)
Volodymyr Mazorchuk (Univ. Uppsala)
Jean-Eric Pin (CNRS/Univ. Paris-Diderot)
John Rhodes (Univ. California, Berkeley)
Emanuele Rodaro (Univ. Porto)
Anne Schilling (Univ. California, Davis)
Benjamin Steinberg (City Univ. New York)
Mária Szendrei (Univ. Szeged)
Marc Zeitoun (Univ. Bordeaux)

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Timetable

	Monday	Tuesday	Wednesday	Thursday	Friday
9:15 - 10:00	Mark Lawson	Victoria Gould	Benjamin Steinberg	Ruy Exel	Volker Diekert
10:00 - 10:20	Vyacheslav Shaprynskii	Itamar Stein	Vladimir Gusev	Parackal G. Romeo	Francesco Matucci
10:20 - 10:40	Arkadiusz Mecel	Karsten Henckell	Célia Borlido	Vojtěch Vorel	Tamás Waldhauser
10:40 - 11:10	Break	Break	Break	Break	Break
11:10 - 11:30	Richard M. Thomas	Manuela Sobral	Lukasz Kubat	Teresa M. Quinteiro	Nasir Sohail
11:30 - 11:50	Arangathu R. Rajan	Andrea Montoli	Filipa Soares	Tatiana B. Jajcayová	Tom Coleman
11:50 - 12:35	Mária Szendrei	José Carlos Costa	Robert Gray	Ganna Kudryavtseva	Anne Schilling
12:35 - 14:15	Lunch	Lunch		Lunch	Lunch
14:15 - 15:00	Marc Zeitoun	John Rhodes		Mark Kambites	João Araújo
15:00 - 15:20	Carlos Ramos	Stuart Margolis		Alonso Castillo-Ramirez	Libor Polak
15:20 - 15:40	Vicente Pérez-Calabuig	Muhammed P. A. Azeef		Saeid Alirezazadeh	Alan Cain
15:40 - 16:10	Break	Break		Break	Break
16:10 - 16:30	Manuel B. Branco	Markus Lohrey (16:10-16:55)	Touristic Tour (16:00-18:30)	Tom Quinn-Gregson	Silke Czarnetzki
16:30 - 16:50	Tara Brough			Hossein Shahzamanian	Jean-Eric Pin (16:30 - 17:15)
16:50 - 17:35	Karl Auinger	Emanuele Rodaro (16:55-17:40)		Volodymyr Mazorchuk	Close
			Conference Dinner (20:00-)		

List of titles

Saeid **Alirezazadeh** .... *Identifying The Structure of The Relatively Free Pro-BSS Forest Algebras*

João **Araújo** .... *The best definition*

Karl **Auinger** .... *On the finite basis problem for deformed diagram monoids and related monoids*

Muhammed P. A. **Azeef** .... *Cross-connections of linear transformation semigroup*

Célia **Borlido** .... *The κ-word problem over pseudovarieties of the form DRH*

Manuel B. **Branco** *On the enumeration of the set of elementary numerical semigroups with fixed multiplicity, Frobenius number or genus*

Tara **Brough** .... *Automaton semigroups: new constructions results and examples of non-automaton semigroups*

Alan **Cain** ... *Combinatorics of cyclic shifts in the plactic, hypoplactic, sylvester, and related monoids*

Alonso **Castillo-Ramirez** .... *Ranks of finite semigroups of cellular automata*

Tom **Coleman** .... *Permutation monoids and MB-homogeneous structures*

José Carlos **Costa** .... *Reducibility of pseudovarieties of the form* $\mathbf{V} * \mathbf{D}$

Silke **Czarnetzki** .... *Logic classes beyond the regular languages described by duality*

Volker **Diekert** .... *Local Rees extensions*

Ruy **Exel** .... *Partial actions and subshifts of infinite type*

Victoria **Gould** .... *Partial semigroups: categories and constellations*

Robert **Gray** .... *Crystal monoids and crystal bases*

Vladimir V. **Gusev** .... *On the interplay between Babai and Černý's conjectures*

List of abstracts

Saeid **Alirezazadeh** *University of Porto*

Identifying The Structure of The Relatively Free Pro-BSS Forest Algebras

Forest algebras are defined for investigating languages of forests [ordered sequences] of unranked trees, where a node may have more than two [ordered] successors [4]. A profinite algebra is defined to be a projective limit of a projective system of finite algebras [2]. For **BSS**[1], the pseudovariety of forest algebras generated by all syntactic forest algebras of piecewise-testable forest languages, we say that a profinite algebra S is pro-**BSS** if it is a projective limit of members of **BSS**.

The natural analog for forest algebras of the structural identification of the relatively free pro-**J** semigroup $\overline{\Omega}_n\mathbf{J}$ as an algebra of type (2,1), see [1,Section 8.2], is to study the relatively free pro-**BSS** forest algebra $\overline{\Omega}_A\mathbf{BSS}$, as an ω-algebra, which retain the equational axioms of forest algebras and are endowed with additional unary operations. In the study of the pseudovariety **BSS**, from [3,Theorem 2 and Proposition 19] and [1, Section 8.2], we obtained certain suitable identities denoted by Σ. We described an algorithm to compute the so-called canonical form for an element of the free ω-algebra $\mathcal{A}$ modulo Σ and we proved it is correct. If the relationship between the free ω-algebra in a certain variety and the free pro-**BSS** algebra is as in the word analog [1,Section 8.2], then the algorithm allows us to identify the structure of the latter. We solved the word problem for the free ω-algebra in the variety $\mathcal{V}$ of ω-algebras defined by the set Σ. Denoting by $F_A\mathcal{V}$ the $\mathcal{V}$-free algebra on A, we then have an ω-algebra homomorphism

$$\varphi : F_A\mathcal{V} \to \overline{\Omega}_A\mathbf{BSS}$$

such that $x_i \mapsto x_i$ $(i = 1, \ldots, n)$.

Speaker in Algebra Workshop 2014-CAUL 34 Years presented a talk: "Forest Algebras, ω-Algebras and A Canonical Form for Certain Relatively Free ω-Algebra" and aimed to identify the structure of the relatively free pro-BSS forest algebras, however it remained open to show that the ω-algebra homomorphism φ is a bijection. In this talk we present a proof for that φ is bijective.

[1] Stands for Bojańczyk, Segoufin, and Straubing as it was first introduced in [3].

References

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João **Araújo** *Universidade Aberta*

The best definition

In this talk I will raise a number of questions and try to provide some answers.
Among others:

(1) What is automated reasoning (AR)?
(2) Has AR solved any important open question?
(3) Is it useful in *real mathematics* or only for equational logic?
(4) Is AR going to put mathematicians out of business?
(5) Is semigroup theory especially fit to AR?
(6) Why AR appears linked with commuting graphs or conjugation in semigroups?
(7) Are there any news coming from the beautiful world that has been knit by permutation groups and transformation semigroups?
(8) In particular, are there any news on synchronization?
(9) It is known for many years that the symmetric group on a finite set X together with one singular map generate all singular transformations on X. What about if we have some group $G < \mathrm{Sym}(X)$?
(10) Are there any news on idempotent generated semigroups?
(11) Is there any link between this talk and its title?

Karl **Auinger** *University of Vienna*

On the finite basis problem for deformed diagram monoids and related monoids

I shall shortly review some of the results of the paper [1], namely a sufficient condition for an (involutory) semigroup to be non-finitely based (NFB) and applications thereof to Kauffman monoids and 'wire monoids'. In the main part of the talk I shall present some new results. These include:

(1) the (Graham–Lehrer) affine Temperley–Lieb monoids ATL_n of rank n are NFB iff $n \geq 3$;
(2) the (Martin–Mazorchuk) monoids MM_n of deformed partitioned binary relations of rank n are NFB iff $n \geq 1$;
(3) the monoids $2Cob(n, n)$ of all 2-cobordisms of rank n are NFB iff $n \geq 1$.

The definition and the structure of these monoids will be also discussed. The NFB-property is enjoyed by the 'plain' monoids as well as by their involutory versions. This is joint work with M. V. Volkov.

References

1. K. Auinger, Y. Chen, X. Hu, Y. Luo, M. V. Volkov, *The finite basis problem for Kauffman monoids*, Algebra Universalis **74** (2015), no. 3-4, 333-350.

Muhammed P. A. **Azeef** *Indian Inst. of Sci. Edu. and Research*

Cross-connections of linear transformation semigroup

The talk will be on the theory of cross-connections with special emphasis on the linear transformation semigroup. In the study of the structure theory of regular semigroups, T E Hall used the principal ideals of the regular semigroup to analyse its structure. P A Grillet refined Hall's theory to abstractly characterize the ideals as *regular partially ordered sets* and constructing the fundamental image of the regular semigroup as a cross-connection semigroup. Later K S S Nambooripad generalized the idea to any arbitrary regular semigroups by characterizing the principal ideals of a regular semigroup as *normal categories*. A cross-connection between two normal categories determines a (regular) cross-connection semigroup and conversely every regular semigroup is isomorphic to a cross-connection semigroup for a suitable cross-connection.

In the talk, I will briefly describe the general cross-connection theory for regular semigroups and use it to study the normal categories arising from the semigroup T_V of singular linear transformations on an arbitrary vectorspace V over a field K. The principal left ideals of T_V are characterized as the category $S(V)$ of proper subspaces of V with linear transformations as morphisms. We show that the semigroup of normal cones in $S(V)$ is isomorphic to T_V. There is an in-built notion of duality in the cross-connection theory; and we observe that it coincides with the conventional algebraic duality of vector spaces. When V is finite dimensional, we show that the normal dual $N^*S(V)$ of $S(V)$ is isomorphic to the category $S(V^*)$ of proper subspaces of V^* where V^* is the algebraic dual space of V. Further we describe various cross-connections between these categories and show that although there are many cross-connections, upto isomorphism, we have only one semigroup arising from these categories. But if we restrict the categories suitably, we can construct some interesting subsemigroups of the *variant* of the linear transformation semigroup. This will provide an alternate way of studying the structure of T_V and also shed light into the more general theory of cross-connections.

Célia **Borlido** *University of Porto*

The κ-word problem over pseudovarieties of the form DRH

The study of finite semigroups is strongly motivated by Eilenberg's correspondence, which establishes a link between varieties of rational languages and pseudovarieties of finite semigroups.

On the level of pseudovarieties, some problems arise naturally, one of them being the so-called "word problem". Roughly speaking, it consists in deciding whether two expressions define the same element in every semigroup of a given pseudovariety V. Let κ denote the canonical implicit signature consisting of two implicit operations: multiplication and $(\omega - 1)$-power. We call κ-word any element of a free κ-semigroup. When the expressions considered in the above mentioned decidability problem are κ-words, then we refer to it as the "κ-word problem over V". One of the pseudovarieties that has been shown to be worth studying is that consisting of all $\mathcal{R}$-trivial semigroups, denoted R. In particular, the κ-word problem over R was proven to be decidable by Almeida and Zeitoun [1]. On the other hand, a natural generalization of R is found in the pseudovarieties of the form DRH for a pseudovariety of groups H. This class consists of all finite semigroups whose regular $\mathcal{R}$-classes are groups from H. Observe that, when H is the trivial pseudovariety, the pseudovariety DRH is nothing but R. Also, the pseudovarieties DRH may be seen as a specialization of the pseudovariety DS, of all finite semigroups whose regular $\mathcal{D}$-classes are subsemigroups. The interest in the latter has been pointed out by Schützenberger [3] in the mid nineteen seventies, through the characterization of the varieties of rational languages corresponding to some subpseudovarieties of DS under Eilenberg's correspondence, among which those of the form DRH. Further, it is worth mentioning the case of the subpseudovariety of DS where the regular $\mathcal{D}$-classes are aperiodic subsemigroups (DA). The corresponding κ-word problem was solved by Moura [2] by adapting the tools used in [1]. We extend the results of [1] by solving the κ-word problem over DRH whenever it is decidable, a property that depends on H. We show that the κ-word problem over DRH may be reduced to the analogous problem for the pseudovariety H. The converse amounts to an easy observation. As it was already mentioned, our approach is inspired by the work of Almeida and Zeitoun [1] on the κ-word problem over R. In order to solve it, they introduced a structure called "R-tree". By putting some additional data on "R-trees", we are able to define "DRH-trees" and use them to characterize the $\mathcal{R}$-classes of the free pro-DRH semigroups. That leads to the claimed reduction of the problem to H.

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Manuel B. **Branco** *University of Évora*

On the enumeration of the set of elementary numerical semigroups with fixed multiplicity, Frobenius number or genus

Let $\mathbb{N}$ denote the set of nonnegative integers. A numerical semigroup is a subset S of $\mathbb{N}$ that is closed under addition, $0 \in S$ and $\mathbb{N} \backslash S$ has finitely many elements. The cardinality of the set $\mathbb{N} \backslash S$ is called the genus of S and it is denoted by $\mathrm{g}(S)$. For any numerical semigroup S, the smallest positive integer belonging to S (respectively, the greatest does not belong to S) is called the multiplicity (respectively Frobenius number) of S and it is denoted by $\mathrm{m}(S)$ (respectively F) (see [6]). We say that a numerical semigroup S is elementary if $\mathrm{F}(S) < 2\mathrm{m}(S)$.

Given a positive integer g, we denote by $\mathcal{S}(g)$ the set of all numerical semigroups with genus g. The problem of determining the cardinal of $\mathcal{S}(g)$ has been widely treated in the literature (see for example [1], [2], [3], [4], [5] and [7]). Some of these works are motivated by Amorós's conjecture [3] which says the sequence of cardinals of $\mathcal{S}(g)$ for $g = 1, 2, \cdots$ has a Fibonacci behavior. It is still not known in general if for a fixed positive integer g there are more numerical semigroups with genus $g + 1$ than numerical semigroups with genus g.

In this talk we give algorithms that allows to compute the set of every elementary numerical semigroups with a given genus g, Frobenius number F and multiplicity m. As a consequence we obtain formulas for the cardinal of these sets. In particular we show that sequence of cardinals of the set of elementary numerical semigroups of genus $g = 0, 1, \ldots$ is a Fibonacci sequence.

[Joint work with: J.C. Rosales (Univ. Granada).]

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Tara **Brough** *University of Lisbon*

Automaton semigroups: new constructions results and examples of non-automaton semigroups

Automaton semigroups are semigroups of endomorphisms of rooted trees generated by the actions of Mealy automata (deterministic synchronous transducers). They act by 'self-similar' endomorphisms, are finitely generated, residually finite and have solvable word problem.

Until recently, only one finitely generated residually finite semigroup had been shown not to be an automaton semigroup, namely $\mathbb{N}$, the free semigroup of rank 1. In this talk I will outline a proof that no subsemigroup of $\mathbb{N}^0$ arises as an automaton semigroup, thus giving an infinite family of residually finite non-automaton semigroups.

I will also give a brief overview of some new ways to build automaton semigroups from known examples, using various standard semigroup constructions such as free products, wreath products, strong semilattices and Rees matrix constructions.

[Joint work with Alan Cain (Univ. NOVA Lisboa).]

Alan **Cain** *Universidade NOVA de Lisboa*

Combinatorics of cyclic shifts in the plactic, hypoplactic, sylvester, and related monoids
The elements of the plactic monoid can be viewed as Young tableaux, and it was proved by Lascoux & Schützenberger in their seminal study [2] that if two of these elements contain the same number of each generating symbol, then one can be transformed to the other by applying a sequence of cyclic shifts (that is, where at each step one moves from an element that factors as xy to the element yx). Thus, if we build a 'cyclic shift graph' whose vertices are elements of the monoid and whose edges connect elements that differ by a cyclic shift, then each connected component consists of precisely those elements that contain a given number of each generating symbol. Choffrut & Mercaş proved that in the plactic monoid of rank n, the number of cyclic shifts required is at most $2n-2$ [1, Theorem 17]. That is, the diameter of a connected component of the cyclic shift graph is at most $2n-2$ (although the number of elements they contain is unbounded).

This talk discusses new results on the cyclic shift graphs for a family of multihomogeneous monoids that, like the plactic monoid, are closely connected with combinatorial objects: the hypoplactic monoid (connected with quasi-ribbon tableaux and quasi-symmetric functions), the sylvester monoid (binary search trees), the taiga monoid (binary search trees with multiplicities), and the stalactic monoid (stalactic tableaux). In each case, the diameter of connected components the cyclic shift graph turns out to be dependent only on the rank of the monoid and not on the number of elements in a connected component. The proofs exploit the combinatorial objects associated to the monoids: other multihomogeneous monoids that have no such associated objects can have unbounded diameters of connected components.

[Joint work with António Malheiro (Centro de Matemática e Aplicações & Departamento de Matemática, Univ. NOVA Lisboa).]

References

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Alonso **Castillo-Ramirez** *Durham University*

Ranks of finite semigroups of cellular automata
Since first introduced by John von Neumann, the notion of cellular automaton has grown into a key concept of computer science, physics and theoretical biology. For any group G and any set A, a cellular automaton over G and A is a transformation $\tau : A^G \to A^G$ (where A^G consists of all maps $x : G \to A$) defined via a finite subset S of G and a local function $\mu : A^S \to A$. The classical and most studied setting is when $G = \mathbb{Z}^d$, $d \in \mathbb{N}$, and A is a finite set; for example, the famous John Conway's Game of Life is a cellular automaton over $G = \mathbb{Z}^2$ and $A = \{0, 1\}$. Recent group theoretic results (see [3]) have motivated the study of cellular automata over various groups, such as amenable and residually finite groups.

Let $(G; A)$ be the set of all cellular automata over G and A. This is a transformation monoid on A^G whose basic semigroup theoretic properties remain unknown. We began the study of some of these properties when G and A are both finite with $|G| = n$ and $|A| = q$, so $(G; A)$ is finite of size q^{q^n}. In this situation, $(G; A)$ turns out to be equal to all the transformations of A^G that commute with the action of G on A^G. Inspired by the results and techniques used in [1], we studied the *rank* (i.e. the size of a smallest generating set) of $(\mathbb{Z}_n; A)$, where $\mathbb{Z}_n$ is the cyclic group of size n (see [2]). We showed that this quantity is intimately related with the divisibility graph of n. We determined the precise rank when $n \in \{2^k, p, 2^k p : p \text{ is an odd prime}, k \geq 1\}$, and found upper and lower bounds for the general case. Some of our results may be generalised for the study of the rank of $(G; A)$, where G is any finite abelian group.

[Joint work with Maximilien Gadouleau (Durham University).]

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Tom **Coleman** *University of East Anglia*

Permutation monoids and MB-homogeneous structures

The study of infinite permutation groups has long been of interest to mathematicians due to its connection to automorphism groups of first order structures. Of particular note are *oligomorphic permutation groups*, which are intimately linked to $\aleph_0$-categorical structures by the famous theorem of Ryll-Nardzewski. *Homogeneous structures* over a finite language provide a rich source of $\aleph_0$-categorical structures and, correspondingly, oligomorphic permutation groups; these structures are characterized by a celebrated theorem of Fraïssé. Since then, complete classifications of homogeneous structures for differing types of relation have been obtained: for posets (Schmerl), graphs (Lachlan and Woodrow) and digraphs (Cherlin).

Group embeddable monoids, by their nature, can be represented as a submonoid of permutations contained in some symmetric group $\mathrm{Sym}(X)$. As every finite group embeddable monoid is a group, we consider infinite submonoids B of the infinite symmetric group $\mathrm{Sym}(\mathbb{N})$ to avoid triviality; such a B is an *infinite permutation monoid*. Natural examples of these occur via the *bimorphism monoid* $\mathrm{Bi}(\mathcal{A})$ of a structure; that is, the collection of bijective endomorphisms of $\mathcal{A}$. It follows that every automorphism of $\mathcal{A}$ is a bimorphism of $\mathcal{A}$ but in general the converse is not true; and so we have that $\mathrm{Aut}(\mathcal{A}) \subseteq \mathrm{Bi}(\mathcal{A}) \subseteq \mathrm{Sym}(A)$, where A is the domain of $\mathcal{A}$.

Recent work in this field by Cameron and Nešetřil [1] and Lockett and Truss [2] generalizes the idea of homogeneity to several notions of *homomorphism-homogeneity.* One such example is the property of *MB-homogeneity*: a structure $\mathcal{A}$ is MB-homogeneous if every monomorphism between finite substructures of $\mathcal{A}$ extends to a bimorphism of $\mathcal{A}$. Analagous to the three classification results above, Lockett and Truss completely classified homomorphism-homogeneous countable posets in [2].

In this talk, connections between permutation monoids and bimorphism monoids of structures are explored in order to develop a notion of oligomorphicity for the case of infinite permutation monoids. In addition to this, a version of Fraïssé's theorem is shown for MB-homogeneous structures, extending work of [1]. Finally, a collection of results is presented on MB-homogeneous graphs; these include constructing $2^{\aleph_0}$ non-isomorphic examples of MB-homogeneous graphs and steps towards a classification result.

[Joint work with David Evans and Robert Gray during the course of my PhD studies at the Univ. East Anglia.]

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José Carlos **Costa** *University of Minho*

Reducibility of pseudovarieties of the form $\mathbf{V} * \mathbf{D}$

The concept of *tameness* of a pseudovariety was introduced by Almeida and Steinberg [2] as a tool for proving decidability of the membership problem for semidirect products of pseudovarieties. The fundamental property for tameness is *reducibility.* This property was originally formulated in terms of graph equation systems and latter extended to any system of equations [1,3]. It is parameterized by an *implicit signature* σ (a set of implicit operations on semigroups containing the multiplication), and we speak of σ-reducibility. For short, given an equation system Σ with rational constraints, a pseudovariety $\mathbf{V}$ is σ-reducible relatively to Σ when the existence of a solution of Σ by implicit operations over $\mathbf{V}$ implies the existence of a solution of Σ by σ-words over $\mathbf{V}$ and satisfying the same constraints. The pseudovariety $\mathbf{V}$ is said to be *σ-reducible* if it is σ-reducible with respect to every finite graph equation system.

This talk is concerned with the κ-reducibility property of semidirect products of the form $\mathbf{V} * \mathbf{D}$, where $\mathbf{D}$ denotes the pseudovariety of all finite semigroups in which idempotents are right zeros and κ is the *canonical signature* consisting of the multiplication and the $(\omega - 1)$-power. We show that, if the pseudovariety $\mathbf{V}$ is κ-reducible, then $\mathbf{V} * \mathbf{D}$ is also κ-reducible.

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[This talk is based on joint work with Conceição Nogueira and M. Lurdes Teixeira.]

Silke **Czarnetzki** *University of Tuebingen*

Logic classes beyond the regular languages described by duality

The study of finite monoids unveiled a rich amount of deep theories that connect over areas from different fields of mathematics.

The well-established connections between pseudovarieties of (finite) monoids, profinite monoids, pseudoidentities and semidirect products seem to form a sound basis for a generalisation of these notions beyond the borders of finiteness. One foundation for a generalisation is laid by the duality established by Stone, of Boolean algebras and topological spaces - so-called Stone spaces. Almeida as well as Pippenger stated that profinite monoids form special instances of Stone spaces. A more recent development by Gehrke, Griegorieff and Pin shows the existence of an extension of pseudoidentities to general Stone spaces. While in the case of profinite monoids, general results are known on how to obtain concrete pseudoidentities, in the case of arbitrary Stone spaces, we lack analogue results.

In an effort to obtain an intuition on how to derive similar notions to obtain identities for the general case, we investigated in Boolean algebras definable by fragments of logic and managed to obtain a sound and complete set of identities for it's Stone Space. These fragments use non-regular predicates, which place them beyond the border of profinite monoids. The used techniques are heavily influenced by the theories developed for pseudoidentities, in which semidirect products play a key role.

Volker **Diekert** *University of Stuttgart*

Local Rees extensions

The talk is based on a joint work with Tobias Walter[1]. It is about finite monoids; and the term *variety* is a shorthand for the more accurate notation pseudovariety.

Jorge Almeida and Ondřej Klíma defined the *bullet operation* $\mathrm{Rees}(\mathbf{U}, \mathbf{V})$ as the least variety of monoids containing all Rees extensions $\mathrm{Rees}(N, L, \rho)$ for $N \in \mathbf{U}$ and $L \in \mathbf{V}$; and they called a variety $\mathbf{V}$ to be *bullet idempotent* if $\mathbf{V} = \mathrm{Rees}(\mathbf{V}, \mathbf{V})$. Their corresponding paper "On the irreducibility of pseudovarieties of semigroups" appeared 2016. One of their results is that $\overline{\mathbf{H}}$ is bullet idempotent. Here $\mathbf{H}$ is any variety of finite groups and $\overline{\mathbf{H}}$ denotes the variety of finite monoids where all subgroups belong to $\mathbf{H}$.

Independently, Tobias Walter and the speaker introduced the notion of *local Rees extension* $\mathrm{Rees}(N, L, \rho)$ of a monoid M, by the restriction that N is a proper submonoid of M, L is a proper local divisor of M, and M itself is a divisor of $\mathrm{Rees}(N, L, \rho)$. In particular, both N and L are smaller than M. This makes the construction useful for induction. The algebraic result for the bullet operation is as follows. Let M be a finite monoid. Then M appears as a divisor of a monoid which is obtained by starting with subgroups of M and applying finitely many steps of local Rees extensions. As a consequence, if $\mathbf{H}$ is any variety of finite groups, then the smallest bullet idempotent variety containing $\mathbf{H}$ is $\overline{\mathbf{H}}$. Phrased differently, the bullet idempotent varieties are exactly the varieties of the form $\overline{\mathbf{H}}$. Thus, a property P holds for all monoids in $\overline{\mathbf{H}}$ if and only if the following two assertions are true.

- Property P holds for all groups in $\mathbf{H}$.
- If P holds for N and L where N is a proper submonoid and L is a proper local divisor of some $M \in \overline{\mathbf{H}}$, then P holds in every divisor of $\mathrm{Rees}(N, L, \rho)$.

[1]Supported by the German Research Foundation (DFG) under grant DI 435/6-1.

Ruy **Exel** *Universidade Federal Santa Catarina*

Partial actions and subshifts of infinite type

A subshift on a finite alphabet Λ is a subset $X \subseteq \Lambda^{\mathbf{N}}$ which is closed in the product topology and invariant under the left shift. A well known result in Symbolic Dynamics asserts that every such X is necessarily given by the set of all infinite words which do not contain any subword of a given set F of *prohibited words*. When X may be described by a finite set of prohibited words, one says that X is a *shift of finite type*. Regarding dynamical properties, shifts of finite type are much better behaved that those of infinite type.

In this talk we will center our attention on O_X, a certain algebra of bounded operators on Hilbert's space associated to a given subshift X. These algebras were first introduced and studied by Matsumoto, with later important contributions by Carlesen.

When X is s subshift of finite type, O_X turns out to be the well known and extensively studied Cuntz-Krieger algebra, but in general the study of O_X presents some very challenging obstacles.

Our approach to the understanding of O_X will be based on the theory of partial group actions and in particular we will show how to describe O_X as the crossed product of a commutative algebra by a natural partial action of the free group.

[This talk is based on joint work with M. Dokuchaev.]

Victoria **Gould** *University of York*

Partial semigroups: categories and constellations

We say that a pair $(P, \cdot)$ is a *partial semigroup* if there is a partial map $P \times P \to P$ such that whenever $(xy)z$ and $x(yz)$ *are both defined*, then $(xy)z = x(yz)$.

Partial semigroups (of various special kinds) have played a significant role in semigroup theory. Biordered sets provide a notable example, where here the domain is determined by a pair of quasi-orders. The partial semigroups we consider here are those underlying *(small) categories*, and *constellations*; again, ordering plays a part, allowing us to extend the partial operation to a global one under certain conditions. Perhaps the best known result of this kind is the Ehresmann-Schein-Nambooripad (ESN) Theorem [4] which shows how to construct an inverse semigroup from a special kind of category called an *inductive groupoid* and moreover, that the category of inverse semigroups is equivalent to the category of inductive groupoids.

Left restriction semigroups are a variety of unary semigroups, modelling the notion of a semigroup of partial maps closed under taking identity maps at domains, and including the class of inverse semigroups. Constellations were introduced by Gould and Hollings [2], who used so-called *inductive constellations* to describe *left restriction semigroups*, thus providing a one-sided version of the ESN Theorem and its extensions.

We observe that left restriction semigroups are a very special case of the *D-semigroups* introduced by Stokes [5]. Within the framework provided by D-semigroups we can define not only left restriction semigroups, but other classes of current interest such as left Ehresmann, left adequate, -unipotent semigroups and their non-regular analogues. Many of these have been studied by Gomes, most recently in [1]: all have left regular band of idempotents.

We survey some extensions of the ESN Theorem and the above result of [2]. We show how, rather surprisingly, 'forgetting' the inductive structure on a constellation allows us to obtain some connections with categories, cementing the idea that a constellation is a one-sided category. Finally we describe some recent work of Stokes [6] that shows how constellations may be used to capture D-semigroups, including those of [1].

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Robert **Gray** *University of East Anglia*

Crystal monoids and crystal bases

The Plactic monoid is a fundamental algebraic object which captures a natural monoid structure carried by the set of semistandard Young tableaux. It arose originally in the work of Schensted (1961) on algorithms for finding the maximal length of a nondecreasing subsequence of a given word over the ordered alphabet $\mathcal{A}_n = \{1 < 2 < ... < n\}$. The output of Schensted's algorithm is a tableau and, by identifying pairs of words that lead to the same tableau, one obtains the Plactic monoid $\mathrm{Pl}(\mathcal{A}_n)$ of rank n. Alternatively, the Plactic monoid may be defined by a finite presentation with generating symbols $\mathcal{A}_n$ and a certain finite set of defining relations which were originally determined in work of Knuth (1970). A third way of obtaining this monoid comes from Kashiwara's theory of crystal bases. The notion of the quantised enveloping algebra, or quantum group, $U_q(\mathfrak{g})$ associated with a symmetrisable Kac–Moody Lie algebra $\mathfrak{g}$ was discovered independently by Drinfeld (1985) and Jimbo (1985) while studying solutions of quantum Yang–Baxter equations. Kashiwara introduced crystals in order to give a combinatorial description of modules over $U_q(\mathfrak{g})$ when q tends to zero. Crystals are useful combinatorial tools for studying representations of these algebras. The vertices of any Kashiwara crystal graph carry a natural monoid structure given by identifying words labelling vertices that appear in the same position of isomorphic components of the crystal. In the special case of Kashiwara crystals of type A_n the monoid that arises from this construction turns out to be the Plactic monoid $\mathrm{Pl}(\mathcal{A}_n)$. In this talk I will present some recent joint work with A. J. Cain and A. Malheiro investigating monoids that arise from Kashiwara crystals in this way. In particular I will discuss the problem of constructing complete rewriting systems, and finding biautomatic structures, for crystal monoids.

Vladimir V. **Gusev** *Université catholique de Louvain and Ural Federal University*

On the interplay between Babai and Černý's conjectures

There are combinatorial problems in group and semigroup theory that can be simply stated, but nevertheless extremely difficult to solve. We focus on two of them, namely, *the Babai conjecture* and *the Černý conjecture*. The former (in the special case of S_n) states that there exists a polynomial $f(n)$ such that for any set of generators G of the full permutation group S_n every permutation from S_n can be represented as a product of at most $f(n)$ elements of G. The latter is a statement about synchronizing automata.
An automaton A is called *synchronizing* if there exist a word w and a state f such that the action of w brings all states to f. Any such word is called *synchronizing* and the length of the shortest synchronizing word is *the reset threshold* of A. The Černý conjecture states that the reset threshold of an n-state automaton is at most $(n-1)^2$.
We introduce a *hybrid Babai-Černý problem*: what are the tight bounds on the reset thresholds of n-state automata with the transition monoid equal to the full transformation semigroup T_n? We present a series of such automata with the reset threshold equal to $\frac{n(n-1)}{2}$. Motivated by our problem we also study 2-transitive automata: for all pairs of states (p_1, q_1) and (p_2, q_2) there exists a word w such that $p_1 \cdot w = p_2$ and $q_1 \cdot w = q_2$.
We construct a series of 2-transitive automata such that for every automaton in the series there exist two pairs of states with the following property: the shortest word that brings one pair to another one has length at least $\frac{n^2}{4} + o(n^2)$.
[Joint work with François Gonze, Balázs Gerencsér, Mikhail V. Volkov and Raphaël M. Jungers (Univ. catholique de Louvain, Alfréd Rényi Institute of Mathematics, and Ural Federal Univ.).]

Karsten **Henckell** *New College of Florida*

Decidability of complexity via pointlike sets: A premodern, elementary and constructive approach

Personal Note:
My aim in this talk is to demonstrate the promise of the "cover-refinement" approach to the complexity problem. I have been working on complexity for over 40 years, and a solution using the "cover-refinement" approach seems to be within reach...yet it has also become clear to me, that without help I will not be able to publish these results. I am therefore extending an open invitation to collaborate (and to co-author) to anyone interested in this approach.
I think the value of results 1 and 2 (A- refinement) basically is tied to the solution of the G-refinement problem (conjectures 3 and 4).
"pre-modern" ,"hyper-elementary"and "explicitly constructive" are not technical terms and allude more to my taste in mathematical tools, and to my desire to give explicit constructions whenever possible.
Some definitions: $\mathbf{A}$ = (pseudovariety of finite) aperiodic semigroups, $\mathbf{G}$ = (pseudovariety of finite) groups, $\mathbf{V_n} = (\mathbf{A} * \mathbf{G})^{\mathbf{n}} * \mathbf{A}$, $P(S)$ = the power set of S, $[S] = \{\{s\} \mid s \in S\}$, $\bigcup : P^2(S) \to P(S)$ is the union - map, $Z(g)$ = the cyclic group generated by (a group element) g, $c(S)$ = minimal n such that $S \in \mathbf{V_n}$ = complexity of S,
$Pl_{\mathbf{V}}(S) = \{X \in S \mid$ for all relations $R : S \to V$ with $V \in \mathbf{V}$ there exists a $v \in V$ such that $X \subseteq R^{-1}(v)\}$.
$Pl_{\mathbf{V}}(S))$ is called "the $\mathbf{V}$ - pointlike sets of S". Then an alternate characterization of $c(S)$ is $c'(S)$ = the minimal n such that $Pl_{\mathbf{V_n}}(S) = [S]$.
Given a relation $R : S \to T$, define $C(R) = \{R^{-1}(t) \mid t \in T\}$, closed under products and subsets $C(R)$ is called "the cover-semigroup presented by R" ; we also say "R computes $C(R)$". If $T \in \mathbf{V}$ we say "$C(R)$ is $\mathbf{V}$ - presentable" or "$C(R)$ is $\mathbf{V}$ - presented" (depending on if we assert the existence of an R, or actually exhibit R). It is well known that $Pl_{\mathbf{V}}(S)$ is $\mathbf{V}$ - presentable. Our goal (the "cover-refinement approach to complexity") is to determine (recursively) $Pl_{\mathbf{W}*\mathbf{V}}(S)$ [for all S], given complete information about $Pl_{\mathbf{V}}(S)$ [for all S]. Ideally, we would accomplish this constructively, i.e. determine a $\mathbf{W} * \mathbf{V}$ - presentation for $Pl_{\mathbf{W}*\mathbf{V}}(S)$ (based on $\mathbf{V}$ - presentations for $Pl_{\mathbf{V}}(S')$ [for various S']).
This ambitious program (the "cover-refinement approach to complexity") has mostly been worked out for the aperiodic case $\mathbf{W} = \mathbf{A}$, while the group case $\mathbf{W} = \mathbf{G}$ is still under investigation.
Part I: the aperiodic case $\mathbf{W} = \mathbf{A}$:
Basic $\mathbf{A}$ -construction Lemma: Let $g \in Pl_{\mathbf{A}*\mathbf{V}}(S)$ be a group element, and let $Z(g) \in Pl_{\mathbf{V}}(Pl_{\mathbf{A}*\mathbf{V}}(S))$, then $\bigcup Z(g) \in Pl_{\mathbf{A}*\mathbf{V}}(S)$ (Note that we need information of $Pl_{\mathbf{V}}$ at $Pl_{\mathbf{A}*\mathbf{V}}(S)$ ... !).
Define $C_{\mathbf{A}}(S)$ = the smallest cover-semigroup C such that $[S] \le C \le P(S)$ and C is closed under
(*) if $g \in C$ is a group element with $Z(g) \in Pl_{\mathbf{V}}(C)$, then $\bigcup Z(g) \in C$.
The Basic $\mathbf{A}$ -construction Lemma then insures that
Theorem 1: $C_{\mathbf{A}}(S) \le Pl_{\mathbf{A}*\mathbf{V}}(S)$
For the opposite direction we adapt our proof (for $\mathbf{V} = \mathbf{1}$) in "Product expansions" (JPAA 101 (1995) pp 157-170) to the general case:
Theorem 2: $Pl_{\mathbf{A}*\mathbf{V}}(S) \le C_{\mathbf{A}}(S)$ (constructively) Note that we actually construct a presentation of $C_{\mathbf{A}}(S)$ from $\mathbf{V}$ - presentations [of various S']
Part II: the group case $\mathbf{W} = \mathbf{G}$:
This is under active investigation. We have a construction $C_{\mathbf{G}}(S)$ and we conjecture:
Conjecture 3: $C_{\mathbf{G}}(S) \le Pl_{\mathbf{G}*\mathbf{V}}(S)$ and
Conjecture 4: $Pl_{\mathbf{A}*\mathbf{G}*\mathbf{A}*\mathbf{V}}(S) \le C_{\mathbf{G}}(S)$ (constructively)
Even though Conjecture 4 is weaker than desired, it is still strong enough to get
Conjecture 5: $c(S)$ is decidable (constructively)

Tatiana **Jajcayová** *Comenius University, Bratislava*

Regular actions of groups and inverse semigroups on combinatorial structures

Every finite group is known to be isomorphic to the automorphism group of some finite graph. This is not the case for specific group actions. In our talk, we address this problem with regard to the most natural group action: the regular action of a group G on itself via multiplication.

We will discuss various combinatorial structures whose full automorphism groups act regularly on their sets of vertices. Equivalently, we attempt to classify finite groups G which admit the introduction of a combinatorial structure on G whose full automorphism group consists solely of the automorphisms induced by the action of the multiplication of G on itself. Such structures can be thought of as combinatorial representations of the corresponding groups.

Previous results on this topic include the classification of graphical regular representations (graphs with regular automorphism groups), classification of digraphical regular representations (directed graphs with regular automorphism groups), as well as the classification of general combinatorial structures (incidence structures) with regular automorphism groups. We generalize these results to the class of k-hypergraphs which are incidence structures with all blocks of size k, and consider the spectrum of all k's for which such representation is possible.

The inverse semigroup of partial automorphisms of a combinatorial structure is a richer and more complicated object that contains more information about the structure than its automorphism group. We propose to study analogous questions to those concerning automorphism groups discussed above for the inverse semigroups of partial automorphisms. The development of such theory has applications in areas of combinatorics that deal with regular objects which are not vertex-transitive.

Mark **Kambites** *University of Manchester*

Combinatorial and geometric amenability-type conditions for semigroups

Within the general theory of amenability (which exists for groups, semigroups and Banach algebras) there is a distinct strand of research focussing on finitely generated groups. The development of a corresponding theory for finitely generated semigroups has been hampered by the lack of an elementary combinatorial description comparable with the *Følner conditions* which characterise amenability in groups. I will discuss some joint research with Robert Gray, on the relationship between amenability, Følner-type conditions and random walks on Cayley graphs of finitely generated semigroups.

Łukasz **Kubat** *University of Warsaw*

Irreducible representations of Chinese monoids

In this talk I will focus on recently obtained results concerning classification of irreducible representations of the Chinese monoid C_n, of any finite rank n, over a field K. It turns out that in case the base field K is uncountable and algebraically closed, all irreducible representations of C_n have a remarkably simple form and they can be built inductively from irreducible representations of the monoid C_2, which are closely related to irreducible representations of the bicyclic monoid. The proof shows also that every such representation of C_n is monomial. Since, as it is already known, C_n embeds into the algebra $K[C_n]/J(K[C_n])$, where $J(K[C_n])$ denotes the Jacobson radical of the monoid algebra $K[C_n]$, a new representation of C_n as a subdirect product of the images of C_n in the endomorphism algebras of the constructed simple modules follows.

[Joint work with Jan Okniński.]

Ganna **Kudryavtseva** *University of Ljubljana*

Stone-type dualities for restriction semigroups

The purpose of this talk is to discuss dualities between some classes of restriction semigroups and respective classes of étale categories (localic or topological). This generalizes dualities between classes of inverse semigroups and respective classes of étale groupoids established earlier by Lawson and Lenz in [2] (topological setting, objects and morphisms) and by Resende in [3] (localic setting, at the level of objects). An important role in our constructions is played by restriction quantal frames, which generalize Resende's inverse quantal frames and capture the multiplicative structure of the frame of opens of the locale C_1 of an étale localic category (C_1, C_0).

Morphisms between étale categories (and between étale groupoids, as a special case) are defined, in the localic setting, simply as the corresponding morphisms of restriction quantal frames, but going in the opposite direction (similarly to the definition of locale maps). In general, these are not frame maps, so that they do not give rise to functors between localic categories. Nevertheless, they can be thought about as some abstract 'relational morphisms', and their image under the spectrum functor to étale topological categories admits a precise description in terms of relational morphisms. Meet preserving morphisms, however, do give rise to functors between localic categories, and are mapped to continuous covering functors when passing to spectrum étale topological categories.

The talk is based on the preprint [1].

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Mark **Lawson** *Heriot-Watt University*

New directions in inverse semigroup theory

It is well-known that inverse semigroups were introduced to provide an algebraic setting for the theory of pseudogroups of transformations. Such pseudgroups played, and continue to play, an important role in defining non-classical geometrical structures and, in addition, also arise in many areas of mathematics such as in group theory. Despite these common roots, inverse semigroup theory and the the theory of pseudogroups of transformations largely went their separate ways. But over the past few years there has been a rapprochement. The aim of my talk is to explain the nature of that rapprochement and to describe the new research that it has inspired.

Markus **Lohrey** *University of Siegen*

Circuit Evaluation for Finite Semirings

Circuit evaluation problems are among the most well-studied computational problems in complexity theory. In its most general formulation, one has an algebraic structure $\mathcal{A} = (A, f_1, \ldots, f_k)$, where the f_i are mappings $f_i : A^{n_i} \to A$. A circuit over the structure $\mathcal{A}$ is a directed acyclic graph (dag) where every inner node is labelled with one of the operations f_i and has exactly n_i incoming edges that are linearly ordered. The leaf nodes of the dag are labelled with elements of A (for this, one needs a suitable finite representation of elements from A), and there is a distinguished output node. The task is to evaluate this dag in the natural way, and to return the value of the output node. If the structure $\mathcal{A}$ is finite, then its circuit evaluation problem can be easily solved in polynomial time, i.e., belongs to the complexity class P. This makes it interesting to characterize the computational complexity of a circuit evaluation problem with respect to the following two complexity classes:

- P-complete problems, i.e. problems $A \in \mathsf{P}$ for which every problem in P can be reduced to A (usually, the reduction is assumed to be logspace computable).
- NC, which is the class of all problems that can be solved in polylogarithmic time with polynomially many processors.

Whereas P-complete problem can be viewed as inherently sequential problems, NC can be viewed as the class of problems that can be efficiently parallelized. Whereas it is clear that $\mathsf{NC} \subseteq \mathsf{P}$, it is a famous open problem in complexity theory, whether this inclusion is strict.

In a paper from 1997, Beaudry, McKenzie, Péladeau, and Thérien studied the circuit evaluation problem for semigroups. They proved the following dichotomy result: If the finite semigroup is solvable (meaning that every subgroup is a solvable group), then circuit evaluation is in NC, otherwise circuit evaluation is P-complete.

In this talk, we will extend the above dichotomy from semigroups to semirings. In a seminal paper from 1975, Ladner proved that the circuit evaluation problem for the Boolean semiring $\mathbb{B}_2 = (\{0,1\}, \vee, \wedge)$ is P-complete. This result marks a cornerstone in the theory of P-completeness. On first sight, it seems that Ladner's result excludes efficient parallel algorithms: One can use it to show that every finite semiring with an additive identity 0 and a multiplicative identity $1 \neq 0$ (where 0 is not necessarily absorbing with respect to multiplication) has a P-complete circuit evaluation problem. Therefore, we take the most general reasonable definition of semirings: A semiring is a structure $(R, +, \cdot)$, where $(R, +)$ is a commutative semigroup, $(R, \cdot)$ is a semigroup, and $\cdot$ distributes (on the left and right) over $+$. In particular, we neither require the existence of a 0 nor a 1. Our main result states that in this general setting there are only two obstacles to efficient parallel circuit evaluation: non-solvability of the multiplicative structure and the existence of a 0 and a $1 \neq 0$ in a subsemiring. More precisely, we show the following result: (i) For every finite semiring $\mathcal{R} = (R, +, \cdot)$, the circuit evaluation problem is in NC if $(R, \cdot)$ is solvable and $\mathcal{R}$ contains no subsemiring with an additive zero 0 and a multiplicative $1 \neq 0$. (ii) Moreover, if one of these conditions fails, then circuit evaluation is P-complete.

The hard part of the proof is to show (i). For this, we first consider the case that the semiring has a 1. For the general case, we reduce the size of the multiplicative subsemigroup generated by the input values of the circuit in one phase, and iterate this process. During this process, we use the previously solved case of circuit evaluation over semirings with a 1 as an oracle.

[Joint work with Moses Ganardi, Danny Hucke, and Daniel König.]

Stuart **Margolis** *Bar-Ilan University*

On the Topology of Boolean Representable Simplicial Complexes

In a series of papers Izhakian and Rhodes introduced the concept of Boolean representation for various algebraic and combinatorial structures. These ideas were inspired by previous work by Izhakian and Rowen on supertropical matrices and were subsequently developed by Rhodes and Silva in a recent monograph, devoted to Boolean representable simplicial complexes (BRSCs).

The original approach was to consider Booloean matrix representations over the Superboolean semiring SB, using appropriate notions of vector independence and rank. Writing $N = \{0, 1, 2, ...\}$, we can define SB as the quotient of $(N, +, ., 0, 1)$ (usual operations) by the congruence that identifies all integers greater or equal to 2. In this context, Boolean representation refers to matrices using only 0 and 1 as entries.

Another approach to Boolean representable simplicial complexes is by means of lattice representations. A simplicial complex is Boolean representable if and only if it equals the set of transversals of the successive differences for chains in some lattice. Precise definitions will be given in John Rhodes's talk.

This talk is devoted to the topology of Boolean representable simplicial complexes. As any finitely presented group can be the fundamental group of a 2-dimensional simplicial complex, the problem of understanding the homotopy type of an arbitrary simplicial complex is hopeless. However, for matroids, the topology is very restricted. Indeed, it is known that a matroid is shellable. This implies that a matroid is homotopy equivalent to a wedge of spheres whose dimension is that of the matroid and rank is a function of its unique non-trivial homology group. In particular, a matroid of dimension at least 2 has a trivial fundamental group.

One of the main results of this talk is to show that the fundamental group of a Boolean representable simplicial complex is a free group. We give a precise formula for the rank of this group in terms of the number and nature of the connected components of its graph of flats.

For 2 dimensional BRSCs, we completely characterize the shellable complexes, showing that these are precisely the sequentially Cohen-Macauley complexes. Although not every 2 dimensional BRSC is shellable, we prove that every 2 dimensional BRSC has the homotopy type of a wedge of 1-spheres and 2-spheres. We consider the connection to EL-labelings of the lattice of flats and give an example of a shellable 2-dimensional complex whose lattice of flats is not EL-labelable .

[Joint work with John Rhodes and Pedro V. Silva (Univ. California, Berkeley and CMUP, Univ. Porto).]

Francesco **Matucci** *University of Campinas*

Decision problems and subgroups in higher dimensional Thompson groups

Higher dimensional Thompson groups nV, first introduced by Matt Brin, are groups of homeomorphisms of powers of the Cantor set. Their description is similar to those of the classical Thompson groups F, T, V but elements present substantial differences, such as having chaotic dynamics. This leads to the existence of undecidable decision problems and makes it harder to work within this group.

In this talk we describe recent results in understanding the dynamics of elements and why it is hard to understand it in most cases and we show that they contain the wide class of right-angled Artin groups as subgroups (which contains, for example, surface groups), leading to further information about decision problems in these groups and recovering another proof that right-angled Artin groups can be realized using asynchronous automata.

[Parts of this work are joint with James Belk, Collin Bleak, Conchita Martinez-Perez and Brita Nucinkis.]

Volodymyr **Mazorchuk** *Uppsala University*

Fiat categorification of IS_n and F_n^*

Ordered monoids (in particular, inverse monoids with respect to the natural order) provide natural examples of 2-categories. However, the asymmetric nature of the partial order usually does not allow one define on such a 2-category any reasonable involution.

In this talk we will show how, starting from the symmetric group S_n, one can construct two 2-categories with involution, the so-called fiat 2-categories. One of them can be viewed as the fiat "extension" of the natural 2-category associated with the symmetric inverse semigroup considered as an ordered semigroup with respect to the natural order. This provides a fiat categorification for the integral semigroup algebra of the symmetric inverse semigroup with respect to the Möbius basis. The other 2-category can be viewed as the fiat "extension" of the 2-category associated with the maximal factorizable subsemigroup of the dual symmetric inverse semigroup (again, considered as an ordered semigroup with respect to the natural order). This 2-category provides a fiat categorification for the integral semigroup algebra of the maximal factorizable subsemigroup of the dual symmetric inverse semigroup.

[This is a report on a joint work with Paul Martin (University of Leeds).]

Arkadiusz **Mecel** *University of Warsaw*

The semigroup of conjugacy classes of left ideals of a finite dimensional algebra

Let A be a finite dimensional unital algebra over a field K. Following [3], I will denote by $C(A)$ the semigroup of conjugacy classes of left ideals of A, equipped with a binary operation induced by the multiplication in A. The general aim is to relate the structure of $C(A)$ with the properties of A. This is a joint work with J. Okniński.

One of the open problems is the description of algebras A with finite $C(A)$. It is strongly related to the classical problem of characterizing the class of algebras of finite representation type [1], [3], especially with the assumption that the base field is algebraically closed. I will present its actual state of progress, especially for the class of radical square zero algebras.

In case when $C(A)$ is finite, certain structural invariants of the algebra A, such as the Gabriel quiver, or the isomorphism class of the algebra $A/J(A)$, can be recovered. This leads to a natural isomorphism problem of characterizing such algebras A, that are determined by $C(A)$, up to isomorphism [2]. Recently obtained examples of such classes will be presented.

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Andrea **Montoli** *University of Coimbra*

The push forward construction and the Baer sum of special Schreier extensions of monoids

The aim for this talk is to describe a Baer sum construction for special Schreier extensions of monoids with abelian kernel. A fundamental tool for this construction is the validity of relative versions of the classical homological lemmas for these extensions, such as the Short Five Lemma [1] and the Nine Lemma [3].

A special Schreier extension $f\colon A \to B$ of monoids, with abelian kernel X, determines a monoid action of B on X. This fact allows to make a partition of the set $\mathrm{SchExt}(B, X)$ of special Schreier extensions of the monoid B by the abelian group X into subsets of the form $\mathrm{SchExt}(B, X, \varphi)$ of special Schreier extensions inducing the same action $\varphi\colon B \to \mathrm{End}(X)$, in analogy with what happens for group extensions.

We first give a description of Baer sums in terms of factor sets [2]: we show that special Schreier extensions with abelian kernel correspond to equivalence classes of factor sets, as it happens for groups. Pointwise multiplication of factor sets induces then an abelian group structure on any set $\mathrm{SchExt}(B, X, \varphi)$.

Secondly, we introduce a push forward construction for special Schreier extensions with abelian kernel [3]. This construction allows us to give an alternative, functorial description of the Baer sum, opening the way to an interpretation of cohomology of monoids in terms of extensions.

[Joint work with Nelson Martins Ferreira and Manuela Sobral.]

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Vicente **Pérez-Calabuig** *University of València*

The soluble kernel of a finite semigroup is computable

The problem of computing kernels of finite semigroups goes back to the early seventies and became popular among semigroup theorists through the Rhodes Type II conjecture which proposed an algorithm to compute the kernel of a finite semigroup with respect to the class of all finite groups. Proofs of this conjecture were given in independent and deep works by Ash and Ribes and Zalesskiĭ, and the results of these authors that led to its proof have been extended in several directions.

Once Rhodes conjecture has been solved, a natural question arising is whether or not the kernel associated to a variety $\mathfrak{F}$ of finite groups is computable. The special case when $\mathfrak{F}$ is the variety of soluble groups is of great importance. The computability of the soluble kernel implies the computability of the pro-soluble closure of a finite rank subgroup, and as Coulbois, Sapir and Weil claimed in 2003, "the solution of this difficult open question would have interesting consequences in finite monoid theory and in computational complexity".

The main aim of this talk is to solve this open question by proving that the soluble kernel of a finite semigroup is computable. Our proof depends heavily on a reduction theorem obtained by us in an earlier paper which shows that the description of the regular elements in the $\mathfrak{F}$-kernel of a semigroup can be resolved by examining the members of a very concrete class of inverse semigroups.

[Joint work with Adolfo Ballester-Bolinches (Univ. València).]

Jean-Eric **Pin** *CNRS/University Paris-Diderot*

Stabilisation monoids and cost functions

Regular cost functions were introduced by Colcombet as a quantitative generalisation of regular languages, retaining many of their equivalent characterisations and decidability properties. For instance, stabilisation monoids play the same role for cost functions as monoids do for regular languages. A *stabilisation monoid* as an ordered monoid M together with a *stabilisation operator* $\sharp : E(M) \to E(M)$ satisfying the following properties:

(S$_1$) for all $s, t \in M$ such that $st \in E(M)$ and $ts \in E(M)$, one has $(st)^\sharp s = s(ts)^\sharp$,

(S$_2$) for all $e \in E(M)$, one has $(e^\sharp)^\sharp = e^\sharp e = ee^\sharp = e^\sharp \leqslant e$,

(S$_3$) for all $e, f \in E(M)$, $e \leqslant f$ implies $e^\sharp \leqslant f^\sharp$,

(S$_4$) $1^\sharp = 1$.

In this lecture, I will first review the main results of this theory. For instance, the standard equivalences on regular languages

regular languages $\iff$ *finite automata* $\iff$ *finite monoids* $\iff$ *monadic second order logic*

admit the following nontrivial extension

regular cost functions $\iff$ *cost automata* $\iff$ *stabilisation monoids* $\iff$ *cost monadic logic*

I will also present two recent results of Daviaud, Kuperberg and the author that extend to cost functions Eilenberg's varieties theorem and profinite equational characterisations of lattices of regular languages.

Libor **Polák** *Masaryk University of Brno*

Graph type conditions on automata determining varieties of languages

• Let $f : B^* \to A^*$ be a morphism, We say that the semiautomaton (no initial nor final states) $(P, B, \circ)$ is an f-*subautomaton* of $(Q, A, \cdot)$ if $P \subseteq Q$ and $q \circ b = q \cdot f(b)$ for every $q \in P$, $b \in B$.

A *variety of semiautomata* $\mathbb{V}$ associates to every finite alphabet A a class $\mathbb{V}(A)$ of semiautomata over alphabet A in such a way that

• $\mathbb{V}(A) \neq \emptyset$ is closed under disjoint unions, finite direct products and morphic images,

• $\mathbb{V}$ is closed under f-subautomata.

For each variety of semiautomata $\mathbb{V}$, we denote by $\alpha(\mathbb{V})$ the class of regular languages given by

$$(\alpha(\mathbb{V}))(A) = \{L \subseteq A^* \mid \exists \mathcal{A} = (Q, A, \cdot, i, F) : L = L_{\mathcal{A}} \text{ and } (Q, A, \cdot) \in \mathbb{V}(A)\}\,.$$

[Ésik and Ito, Chaubard, Pin and Straubing] The mapping α is an isomorphism of the lattice of all varieties of semiautomata onto the lattice of all varieties of regular languages. But how to present varieties of semiautomata? One of several possibilities follows.

Concrete examples of Forbidden Patterns characterizations

• In [J.-E. Pin '92] the author proves a characterization for reversible languages.

• In [J. Cohen, D. Perrin and J.-E. Pin '93] such characterizations of the syntactic semigroup being R-trivial, locally R-trivial, L-trivial, and locally L-trivial are given.

• In [J.-E. Pin, P. Weil '97] the authors succeeded to characterize the cases the classes 1/2 and 3/2 of the Straubing-Thérien hierarchy, 1/2 of the dot-depth hierarchy and the level 1 of the group hierarchy.

• In [Sz. Iván, J. Nagy-Gyorgy '14] the authors deal with finite and cofinite, definite, reverse definite and generalized definite languages. (Some of the results were proved elsewhere.)

• In [J.-E. Pin, the book '16] the author deals with slender and sparse languages.

As usual, one tests the conditions in the minimal complete DFA for a given language. We will present quite general theory of Forbidden Patterns for characterizations of classes of languages.

Tom **Quinn-Gregson** *University of York*

Homogeneous bands

The concept of homogeneity of relational structures has connections to model theory, permutation groups and combinatorics. A number of complete classifications of homogeneous structures have been obtained, including those for graphs, semilattices and posets. We may naturally extend the definition of homogeneity to algebras, by defining an algebra to be homogeneous if every isomorphism between finitely generated sub-substructures extends to an automorphism. The key to this extension is that connections with model theoretic properties such as Quantifier Elimination and ω-categoricity remain.
A band is a semigroup in which every element is an idempotent, that is, if $a \in S$ then $a^2 = a$. Every band is equipped with a partial order, given by $e \leq f$ if and only if $ef = fe = e$. In this talk I will discuss when the homogeneity of a band passes to the poset it induces, and give a complete classification in this case.

Teresa M. **Quinteiro** *Inst. Sup. Eng. Lisboa*

Bilateral decompositions of some monoids of transformations

Let S and T be two semigroups. Let

$$\begin{array}{rlllllll} \delta: & T & \longrightarrow & \mathcal{T}(S) & & & \\ & u & \longmapsto & \delta_u: & S & \longrightarrow & S \\ & & & & s & \longmapsto & u\cdot s \end{array} \quad \text{and} \quad \begin{array}{rlllllll} \varphi: & S & \rightarrow & \mathcal{T}(T) & & & \\ & s & \longmapsto & \varphi_s: & T & \longrightarrow & T \\ & & & & u & \longmapsto & u^s \end{array}$$

be an anti-homomorphism of semigroups and a homomorphism of semigroups, respectively, such that

: (SPR) $(uv)^s = u^{v\cdot s}v^s$, for $s \in S$ and $u, v \in T$ (*Sequential Processing Rule*) and

: (SCR) $u \cdot (sr) = (u \cdot s)(u^s \cdot r)$, for $s, r \in S$ and $u \in T$ (*Serial Composition Rule*).

Within these conditions, the set $S \times T$ is a semigroup with respect to the multiplication $(s, u)(r, v) = (s(u \cdot r), u^r v)$, for $s, r \in S$ and $u, v \in T$. We denote this semigroup by ST and call it the *(Kunze) bilateral semidirect product* of S and T (associated with δ and φ). If S and T are monoids and δ and φ are *monoidal* (i.e. $u \cdot 1 = 1$, for $u \in T$, and $1^s = 1$, for $s \in S$) and preserve the identity, then ST is a monoid with identity $(1, 1)$.
This notion was introduced and studied by Kunze and was strongly motivated by automata theoretic ideas.
In this talk we will present decompositions of several monoids of transformations by means of bilateral semidirect products and quocients.
[Joint work with Vítor Hugo Fernandes (Univ. NOVA Lisboa).]

Arangathu R. **Rajan** *University of Kerala*

Categories determined by inverse semigroups

Several categories arise in the structure theory of inverse semigroups and also of regular semigroups. For example the well known structure theorem of Schein for inverse semigroups is based on the inductive groupoid associated with the inverse semigroup. Here a groupoid is a category in which all morphisms are isomorphisms. Several other categories associated with inverse semigroups can be seen in literature, for example in the works of J.E. Pin and Stuart Margolis, MV Lawson, AR Rajan etc.
One category of interest in arising in the study of structure of regular semigroups is the category of principal left [right] ideals of the semigroup introduced by K.S.S. Nambooripad. These categories have been abstractly characterised as normal categories. These are categories in which the objects are principal left [right] ideals of a regular semigroup and morphisms are certain right[resp. left] translations. Normal categories are the basic categories considered in the cross connection theory of regular semigroups, which is a generalization of Grillet's cross connection. One interesting question associated with study of normal categories is determination of normal categories arising from various classes of regular semigroups. We consider this question for certain classes of inverse semigroups and describe the special properties of the corresponding normal categories.

Carlos **Ramos** *University of Évora*

Recombination algebraic structure for Cellular Automata

We define and study an algebraic structure arising from recombination processes for Cellular automata. Cellular automata are discrete dynamical systems, with $\mathbb{Z}_n$ as local state space, and each cellular automaton is characterized by a finite sequence of elements in $\mathbb{Z}_n$, determining the time evolution rule. This sequence is seen as the genotype of the cellular automata. Therefore, the space of cellular automata of n local states can be seen as the set of finite sequences, of certain size m, in $\mathbb{Z}_n$, that is $\mathbb{Z}_n^m$. Recombination in cellular automata is a process analogue to the occurring in the DNA of living beings through the reproduction process. We have defined in [1] a suitable binary operation which produces the recombination of two finite sequences. This operation, $\circ_\alpha$, is parametrized by real number α in the unit interval. Therefore, fixed n and m, we obtain a one parameter family of algebraic structures, with set $\mathbb{Z}_n^m$ and operation $\circ_\alpha$. This algebraic structure has certain properties such as non-commutativity and non-associativity. The main objective of our work is to study the algebraic structure generated by a finite initial population of cellular automata through recombination operation. In particular, to study the Cayley graph analogue for our structure, which can be seen as the phylogenetic tree for the initial population. The question is, depending on α, what is the maximal diversity in the genotypes? What is changing in the algebraic structure as we change α?

[Joint work with António Patrício, Ana Paula Pimenta (FCT, Univ. NOVA Lisboa).]

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John **Rhodes** *University of California, Berkeley*

Boolean representations of simplicial complexes: beyond matroids

We give a "crash course" on finite simplicial complexes admitting Boolean representations. Arbitrary finite matroids arise as a particular case. Zur Izhakian and I invented the theory of Boolean representations of simplicial complexes in 2008, and Pedro Silva and I later developed and matured the theory (see *Boolean Representations of Simplicial Complexes and Matroids*, by Rhodes and Silva, Springer Monographs in Mathematics, 2015). Stuart Margolis has recently made contributions to the theory. A background in standard undergraduate mathematics (linear algebra and combinatorics) is all that would be required to understand this talk.

Emanuele **Rodaro** *Politecnico di Milano*

Equalizers and kernels in categories of monoids

Let Mon be the category of monoids and C a full subcategory of Mon. The first aim of this seminar is to study equalizers in the category C when C is, in particular, one of the categories Mon, CMon, cCMon, rKMon, IMon and of all monoids, commutative monoids, cancellative commutative monoids, reduced Krull monoids, inverse monoids and free monoids, respectively. In all these categories C, the equalizer of any two morphisms $f, g\colon M \to N$ in C exists and has the form $\varepsilon\colon E \to M$, where $E = \{x \in M \mid f(x) = g(x)\}$ and ε is the embedding (in order to see this, the only thing that must be checked is that if f and g are morphims in C, then the submonoid E of M is an object of C).

For a monoid morphism $f\colon M \to N$ between two monoids M and N, the natural notion of kernel is that of the congruence $\sim_f$ on M defined, for every $m, m' \in M$, by $m \sim_f m'$ if $f(m) = f(m')$. Here we will use the completely different notion of kernel in the categorical sense. Indeed, since all these categories C contain the trivial monoid (with one element), which is the zero object in C, a second related problem we consider is that of determining the kernels in C, i.e., the equalizers of a morphism in C and the zero morphism. In all the aforementioned categories, the kernel of $f\colon M \to N$ is simply the embedding of the submonoid $f^{-1}(1_N)$ into M, but a complete characterization of kernels in these categories is not always trivial, and leads to interesting related notions. Kernels (in the categorical sense) and equalizers identify interesting classes of monomorphisms.

As a motivation, consider for instance, the well known cases in which C is one of the categories Grp, Ab or TF of all groups, abelian groups, torsion free abelian groups, respectively. In these categories, the concepts we run into when we determine equalizers and kernels are those of normal subgroup and pure subgroup. Hence we determine kernels and equalizers in subcategories C of Mon to identify analogues of normal subgroups and pure subgroups in the case of the categories Mon, CMon, cCMon, rKMon, IMon and .

[Joint work with Alberto Facchini (University of Padova).]

Parackal G. **Romeo** *Cochin University of Science and Technology*

On the Lattice of Biorder Ideals of Regular Rings

Biordered set was introduced by Nambooripad to describe the structure of the set of idempotents of a semigroup, which we recall below. For the properties and results regarding biordered sets see [5].

A partial algebra E is a set together with a partial binary operation on E. Then $(e, f) \in D_E$ if and only if the product ef exists in the partial algebra E. On E we define

$$\omega^r = \{(e, f) : fe = e\} \quad \omega^l = \{(e, f) : ef = e\}$$

$$\omega^r(e) = \{f : ef = f\} \quad \omega^l(e) = \{f : fe = f\}$$

and $\mathcal{R} = \omega^r \cap (\omega^r)^{-1}$, $\mathcal{L} = \omega^l \cap (\omega^l)^{-1}, and\ \omega = \omega^r \cap \omega^l$. We will refer ω^r and ω^l as the right and the left quasiorder of E.

Definition 1. *Let E be a partial algebra. Then E is a biordered set if the following axioms and their duals hold:*

(1) ω^r and ω^l are quasi orders on E and

$$D_E = (\omega^r \cup \omega^l) \cup (\omega^r \cup \omega^l)^{-1}$$

(2) $f \in \omega^r(e) \Rightarrow \mathcal{R}\ fe\ \omega\ e$

(3) $g\ \omega^l\ f$ and $f, g \in \omega^r(e) \Rightarrow ge\ \omega^l\ fe$.

(4) $g\ \omega^r\ f\ \omega^r\ e \Rightarrow gf = (ge)f$

(5) $g\ \omega^l\ f$ and $f, g \in \omega^r(e) \Rightarrow (fg)e = (fe)(ge)$.

We write $E = \langle E, \omega^l, \omega^r\rangle$ to mean that E is a biordered set with quasiorders ω^l, ω^r. Let $\mathcal{M}(e, f)$ denote the quasi ordered set $(\omega^l(e) \cap \omega^r(f), <)$ where $<$ is defined by $g < h \Leftrightarrow eg\ \omega^r\ eh$, and $gf\ \omega^l\ hf$. Then the set

$$S(e, f) = \{h \in M(e, f) : g < h \text{ for all } g \in M(e, f)\}$$

is called the sandwich set of e and f.

(1) $f, g \in \omega^r(e) \Rightarrow S(f, g)e = S(fe, ge)$

The biordered set E is said to be regular if $S(e, f) \neq \emptyset$ for all $e, f \in E$.

Definition 2. *For $e \in E$, $\omega^r(e)$ $\left[\omega^l(e)\right]$ are principal right [left] ideals and $\omega(e)$ is a principal two sided ideal and are called biorder ideals generated by e.*

Principal Ideals of Regular Ring

A ring $(R, +, \cdot)$ is called regular if for every $a \in R$ there exists an element a' such that $aa'a = a$. A subset A of a ring $\mathcal{R}$ is called right ideal in case

$$x + y \in A, \ xz \in A$$

for each $x, y \in A$ and $z \in \mathcal{R}$.

If R is a ring and $\mathbf{a} \subset \mathbf{R}$ is a right ideal then $\mathbf{a}$ has a unique least extension $\langle a\rangle_r$ containing $\mathbf{a}$. Similarly we have the unique left ideal $\langle a\rangle_l$ and two sided ideal $\langle a\rangle$ containing $\mathbf{a}$.

Definition 3. *A principal right [left] ideal is one of the from $\langle a\rangle_r[\langle a\rangle_l]$. The class of all principal right [left] ideals will be denoted by $\bar{R}_{\mathcal{R}}\ [\bar{L}_{\mathcal{R}}]$.*

Theorem 1. *Let R be a regular ring, then the set $\bar{R}_{\mathcal{R}}$ is a complemented, modular lattice partially ordered by $\subset$, the meet being $\cap$ and join $\cup$, its zero is $\langle 0\rangle_r$ and its unit is $\langle 1\rangle_r$.*

Analogous to von Neumann's construction of the principal ideals of a regular ring, the structure of the biorder ideals of regular rings are described in the following theorems.

Theorem 2. *Let R be a ring then the set of all principal ω^l-ideals Ω_L and the set of all principal ω^r-ideals Ω_R of R are complemented, modular lattices ordered by the relation $\subset$, the meet being $\cap$ and the join $\cup$; its zero is 0, and its unit is $\omega^l(1)[\omega^r(1)]$.*

Theorem 3. *Let R be regular ring with $M(e_i, e_j) = \{0\}$ for $i \neq j$ and $d_l(e_i, e_j) \leq 3$. Then the complemented, modular lattice $\Omega_L[\Omega_R]$ is of order n.*

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Anne **Schilling** *University of California, Davis*

A Markov chain on semaphore codes and the fixed point forest

We present two combinatorial/probabilistic models that might be of interest to the study of semigroups: The first structure is a Markov chain on semaphore codes that appeared in joint work with Pedro Silva and John Rhodes (arXiv:1509.03383 and arXiv:1604.00959, IJAC to appear). A semaphore code is a suffix code with a right action of the semigroup $A^{\leq k}$ of words of length at most k in the alphabet A. Multiplication in this semigroup is concatenation and taking the last k letters if the length is bigger than k. It turns out that semaphore codes approximate right congruences in the lattice (by inclusion) of right congruences. An important question is which elements in the semigroup are resets, that is, act as constant maps on a right congruence. On semaphore codes, we are able to compute the stationary distribution and hitting time to reset explicitly.

Elliptic maps on finite trees provide a powerful tool to study semigroups. Hence an understanding of properties of trees is important. In her undergraduate thesis at UC Davis in 2015, Gwen McKinley studied the following partial sorting algorithm on permutations of size n. Take the first entry in the one-line notation of the permutation and move it into the place of its value. This gives rise to a forest structure, which we call fixed point forest, with derangements as leaves and permutations π with $\pi(1) = 1$ as roots. Despite its simple description it exhibits a rich structure. In joint work with Tobias Johnson and Erik Slivken (arXiv:1605.09777) we analyze the local structure of the tree at a random permutation in the limit as $n \to \infty$.

Hossein **Shahzamanian** *University of Porto*

The rank of variants of nilpotent pseudovarieties

Mal'cev and independently Neumann and Taylor have shown that nilpotent groups can be defined by semigroup identities (that is, without using inverses). This leads to the notion of a nilpotent semigroup (in the sense of Mal'cev). The finite nilpotent semigroups, the finite Neumann-Taylor semigroups, the finite positively Engel semigroups and the finite Thue-Morse semigroups, separately, constitute pseudovarieties. These are examples of ultimate equational definitions of pseudovarieties in the sense of Eilenberg and Schützenberger. We denote them, respectively, by MN, NT, PE and TM. We investigated the rank of these pseudovarieties. We showed that the pseudovariety NT has infinite rank and, therefore, it is non-finitely based.

Finite aperiodic (in the sense of Mal'cev) semigroups can be divided in four classes. The pseudovariety $\mathsf{BG_{nil}}$ does not contain any semigroup in the class (1). The pseudovariety PE does not contain any semigroup in the classes (1) and (2) and the pseudovariety MN contains semigroups in none of them. On the other hand, $S \in \mathsf{PE}$ if $S \in \mathsf{BG_{nil}}$ and $F_7 \not\prec S$. We introduced the pseudovariety . A finite semigroup S is in if $S \in \mathsf{BG_{nil}}$, $F_7 \not\prec S$ and $F_{12} \not\prec S$. We proved that semigroups constitutes a pseudovariety. The pseudovariety is strictly contained in the pseudovariety PE and strictly includes the pseudovariety NT and it does not contain any semigroup in the classes (1), (2) and (3).

[Joint work with Jorge Almeida (CMUP, Univ. Porto).]

Vyacheslav **Shaprynskii** *Ural Federal University*

An example of non-nilpotent almost nilpotent nilsemigroup

A semigroup is called *almost nilpotent* if each its proper subsemigroup is nilpotent. Investigation of almost nilpotent semigroups was inspired by Lev Shevrin since the early sixties. Describing almost nilpotent semigroups which are not nilsemigroups is nearly trivial. The nil-case appeared to be much more difficult. The following problem was stated in [1]:

Question. Is every almost nilpotent nilsemigroup nilpotent?

This problem was the subject of a series of papers. The results obtained there are observed in the survey [2]. In the same article a candidate for being a counter-example is suggested. This semigroup may be defined by an infinite set of relations over a countably infinite alphabet. That relations are chosen in such a way that either the resulting semigroup is a counter-example or the relations collapse this semigroup into one element. Unfortunately, A.N.Silkin proved at the beginning of 1990's that the latter is the case. The aim of my report is to present a modified version of the semigroup in [2] which avoids the mentioned collapsing effect. This result completes the search for a counter-example.

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Filipa **Soares** *Inst. Sup. Eng. Lisboa*

Local finiteness for Green relations in (I-)semigroup varieties

This work is devoted to the notion of local $\mathcal{K}$-finiteness for varieties, where $\mathcal{K}$ stands for any of the five Green's relations, with respect to varieties of semigroups and *varieties of I-semigroups*, which are classes of algebras of type $(2,1)$ satisfying the identities

$$x(yz) = (xy)z\,,\ xx'x = x\,,\ (x')' = x$$

(with $a \mapsto a'$ denoting the unary operation). Thus, for instance, both completely regular semigroup varieties and inverse semigroup varieties can be found within varieties of I-semigroups.

For $\mathcal{K} \in \{\mathcal{H}, \mathcal{L}, \mathcal{R}, \mathcal{D}, \mathcal{J}\}$, we say that a semigroup is *$\mathcal{K}$-finite* if it contains only finitely many (distinct) $\mathcal{K}$-classes and that a variety $\mathbf{V}$ of (I-)semigroups is *locally $\mathcal{K}$-finite* if every finite generated semigroup from $\mathbf{V}$ is $\mathcal{K}$-finite. Thus, these notions constitute a generalization of the important concept of locally finite variety.

In this talk, several properties of $\mathcal{K}$-finite semigroups will be described and the lattices of varieties of semigroups and of varieties of I-semigroups characterised with respect to these properties. Namely, we address the connections between $\mathcal{H}$-, $\mathcal{L}$-, $\mathcal{R}$-, $\mathcal{D}$-, and $\mathcal{J}$-finiteness, conservation or loss under certain operators, and apply them to the study of varieties. The present success of the characterisation varies; if the case of completely regular semigroup varieties is fairly settled, some questions remain unanswered for inverse semigroup varieties, and many persist in semigroup varieties.

[Joint work with Pedro V. Silva (CMUP, Univ. Porto).]

Manuela **Sobral** *University of Coimbra*

Homological lemmas for Schreier extensions of monoids

In the paper [3] we introduced the notion of Schreier split epimorphism of monoids. These split epimorphisms correspond to classical monoid actions, that is monoid homomorphisms into the monoid of endomorphisms, recovering in this context the classical equivalence between group actions and split extensions. Such equivalence allowed to obtain a description of crossed modules in terms of internal structures [5,3].

Later, in [1,2], we investigated some Mal'tsev-type properties of Schreier split epimorphisms. Namely, we introduced the notion of Schreier reflexive relation and proved that all such relations are transitive, being equivalence relations if and only if their zero-classes are groups. This lead to the notion of special Schreier extension: a surjective monoid homomorphism f is a special Schreier extension when its kernel congruence is a Schreier equivalence relation. This amounts to have a partial subtraction on the domain of f.

In this talk, we will show that special Schreier extensions satisfy relative versions of the classical homological lemmas, such as the (Split) Short Five Lemma [1,2] and the Nine Lemma [4]. These are the building blocks to construct Baer sums of special Schreier extensions with abelian kernel.

[Joint work with Nelson Martins Ferreira and Andrea Montoli.]

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Nasir **Sohail** *Wilfrid Laurier University*

On Epimorphisms of Ordered Algebras

Flatness properties and amalgamation of monoids are known to undergo 'sever' restrictions if a compatible order is introduced on top of the algebraic structure. For instance the three-element chain semilattice, that is absolutely flat and hence an amalgamation base in the class of all monoids, fails to retain these properties in the ordered context for its nine out of thirteen compatible orders. We, however, proved in 2015 that epimorphisms of monoids (and semigroups), are not affected by the introduction of order. In this talk we shall present our recent work which shows that epimorphisms in certain varieties of algebras may not be affected by the introduction of order. In particular, considering the varieties of all ordered and unordered algebras of a given type we show that epimorphisms are surjective in both of these varieties. We also show that in varieties defined via 'balanced identities' epimorphisms are not affected by order.

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Itamar **Stein** *Bar Ilan University*

Algebras of Ehresmann semigroups and categories

The well known Ehresmann-Schein-Nambooripad (ESN) theorem states that the category of inverse semigroups is isomorphic to the category of inductive groupoids with inductive functors as morphisms. E-Ehresmann semigroups are a commonly studied generalization of inverse semigroups. There is also a notion of an Ehresmann category which is a generalization of an inductive groupoid. Lawson generalized the ESN theorem and proved that the category of all E-Ehresmann semigroups is isomorphic to the category of all Ehresmann categories. In this talk we will show that under some finiteness condition, the semigroup algebra of an E-Ehresmann semigroup is isomorphic to the category algebra of the corresponding Ehresmann category. This generalizes a result of Steinberg who proved this isomorphism for inverse semigroups and inductive groupoids and a result of Guo and Chen who proved it for finite ample semigroups. We also characterize E-Ehresmann semigroups whose coresponding Ehresmann category is an EI-category, i.e., a category for which every endomorphism is an isomorphism. The case of Ehresmann EI categories is interesting because in this case there is a way to describe the Jacobson radical and the ordinary quiver of the category algebra. We will give some natural examples and show that in certain cases the maximal semisimple image of the semigroup algebra is spanned by an inverse subsemigroup of the E-Ehresmann semigroup.

Benjamin **Steinberg** *City University of New York*

Model theory and the free pro-aperiodic monoid

We introduce an approach to the free pro-aperiodic monoid using model theory. This method allows one to apply arguments very similar to those of combinatorics on words to elements of the free pro-aperiodic monoid. We recover many of the known results about the algebraic structure of this monoid in a fairly elementary way (once one absorbs some classical model theory). The method is particularly effective for omega-terms, where we are able to obtain a number of results of Almeida, Costa, Zeitoun and of Huschenbett and Kufleitner that previously relied on McCammond's normal form theorem, which arose out of his work on free Burnside semigroups, as well as generalizations.

This talk will assume no background on model theory and very little background on profinite semigroups.
[Joint work with my postdoc Sam van Gool.]

Mária **Szendrei** *University of Szeged*

Embedding in factorisable restriction monoids

Restriction semigroups have arisen from a number of mathematical perspectives. They are semigroups equipped with two additional unary operations which satisfy certain identities, and they are non-regular generalisations of inverse semigroups. Each inverse semigroup determines a restriction semigroup where the unary operations assign the idempotents aa^{-1} and $a^{-1}a$, respectively, to any element a. The class of restriction semigroups is just the variety of algebras generated by these restriction semigroups obtained from inverse semigroups [1].

So far, a number of important results of the structure theory of inverse semigroups have been recast in the broader setting of restriction semigroups. It is established that each restriction semigroup has a proper cover where a proper restriction semigroup is the analogue of an E-unitary inverse semigroup [1]. Notions of factorisability and almost factorisability — both one- and two-sided versions — are introduced in a way similar to the inverse case [2], [4]. Moreover, each of these restriction semigroups is characterised as a (projection separating) homomorphic image of a semidirect-like product of a semilattice by a monoid. In particular, a restriction monoid turns out to be factorisable if and only if it is a (projection separating) homomorphic image of a semidirect product of a semilattice monoid by a monoid where the latter monoid acts on the semilattice monoid by automorphisms. Finally, each restriction semigroup is proved to be embeddable in an almost left factorisable restriction semigroup [4].

Since the definition of a restriction semigroup is left-right symmetric, it is natural to ask whether a stronger result holds where one-sided factorisability is replaced by two-sided. The main result of the talk answers this question in the affirmative:

Theorem. *Each restriction semigroup is embeddable in a factorisable restriction monoid.*
[Joint work with Victoria Gould and Miklós Hartmann [3].]

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Richard M. **Thomas** *University of Leicester*

Word problems and formal language theory

The word problem of a finitely generated group is a fundamental notion in group theory; we choose a finite generating set for our group G and then define the word problem of G to be the set of all the words in the generators of the group that represent the identity element of G. This formulation allows us to consider the word problem of a group as a formal language and there has been considerable research concerning the connections between the complexity of this set of words as a formal language and the algebraic structure of the corresponding group.

One interesting question is that of asking, given a particular family $\mathcal{F}$ of formal languages, which groups G have a word problem lying in $\mathcal{F}$. It would appear that whether or not the word problem of a group G lies in the family $\mathcal{F}$ depends on the choice of generating set for G, but it is well known that this is not generally the case for natural families of languages.

Another interesting question is that of characterizing which languages are word problems of groups, asking, in particular, what sets of conditions on languages are necessary and sufficient for that language to be a word problem of a finitely generated group. A related question is that of the decidability of such conditions for certain natural families $\mathcal{F}$ of languages.

A natural question is the extent to which this generalizes to finitely generated semigroups. In a group G two words u and v over the generators represent the same element of G if and only if uv^{-1} represents the identity element, which is why we focus on the set of words representing the identity in that case, but this will no longer work when we consider semigroups. Following Duncan and Gilman one could define the word problem of a finitely generated semigroup S to be the set of all words of the form $u\#v^{rev}$ where u and v are words in the generators of S which represent the same element of S; here $\#$ is a new symbol that is not a generator of S and v^{rev} denotes the reversal of the word v.

Given this we can talk about the word problem of a finitely generated semigroup lying in a family $\mathcal{F}$ of formal languages as well. As with groups, the membership of the word problem of a semigroup S lying in $\mathcal{F}$ is independent of the choice of finite generating set for S under certain mild assumptions on the family $\mathcal{F}$. This definition for semigroups is a natural extension of the notion of the word problem from groups to semigroups since the word problem of a group G in the group sense lies in a family $\mathcal{F}$ of languages if and only if the word problem of G in the semigroup sense lies in $\mathcal{F}$.

The purpose of this talk is to survey some of what is known about these problems and to mention some open questions. We will be focussing on families of languages low in the Chomsky hierarchy, namely the regular, one-counter and context-free languages.

Uncertainty and Synchronization

In a system described by a deterministic finite automaton $A = (Q, \Sigma, \delta)$, uncertainty corresponds to a set of *possible states* $S \subseteq Q$. A *reset word* is an input sequence $w \in \Sigma^*$ that maps all the states of S to a single state, i.e., $\|\delta(S, w)\| = 1$. The process of modifying and reducing the current uncertainty by applying the letters of w is referred to as *synchronization*. We ask:

(1) For a given DFA with a given uncertainty, does there exist a reset word?
(2) For a given DFA with a given uncertainty, what is the minimum length of reset words?
(3) For a given n, what is the greatest minimum length between all n-state DFA?

The special case of $S = Q$ is widely studied within the pursuit of resolving the Černý conjecture. In that case, shortest reset words are at most of cubic length in the number of states and their existence can be tested in polynomial time. In the general scope of $S \subseteq Q$, the lengths of shortest reset words become exponential and the testing becomes PSPACE-complete. Though these facts became classical during the last century, the field was not explored with enough precision. This contribution presents recent results that give answers to the following key questions:

(1) Is there a polynomial (or at least $2^{o(n)}$) upper bound on the length of shortest reset words in strongly connected n-state DFA?
(2) Is there a $2^{o(n)}$ upper bound on the length of shortest reset words in n-state DFA with a fixed alphabet?

Note that $2^{\mathcal{O}(n)}$ is a general upper bound following from the number of possible uncertainties. Unfortunately, both the above questions turn out to have negative answers. The new $2^{\Omega(n)}$ lower bound involves DFA that combine both the restrictions, i.e., are strongly connected and binary [5]. The following table shows that in certain sence the new result closes the history of lower bounds on minimum lengths of reset words.

	alphabet size	**strong connectivity**	**min. length of reset words**
Subset listing construction [1]	$2^{\theta(n)}$	no	$2^{\theta(n)}$
Basic radix construction [3]	$\theta(n)$	no	$2^{\theta(n)}$
High-order permutation constr. [2]	2	no	$2^{\theta\left(\sqrt[3]{n}\log n\right)}$
Extended radix construction [4]	2	no	$2^{\theta\left(\frac{n}{\log n}\right)}$
The new method [5]	2	yes	$2^{\theta(n)}$

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Tamás **Waldhauser** *University of Szeged*

Strong affine representations of the polycyclic monoids

Certain representations of the polycyclic monoid $\mathcal{P}_n$ and of the Cuntz C^* algebra $\mathcal{O}_n$ can be given by so-called branching function systems $(X; f_1, \ldots, f_n)$, where

- X is an infinite set,
- each $f_i \colon X \to X$ is an injective function,
- the sets $f_i(X)$ are pairwise disjoint, and
- $X = f_1(X) \cup \cdots \cup f_n(X)$.

We get a very natural special case by letting $X = \mathbb{Z}$ and $f_i(x) = nx + d_i$, where $d_1, \ldots, d_n$ is a complete system of residues modulo n. The union of the inverse maps gives a transformation $R := f_1^{-1} \cup \cdots \cup f_n^{-1}$ of $\mathbb{Z}$, yielding a discrete dynamical system $(\mathbb{Z}; R)$. The periodic points of this dynamical system (called atoms in the context of representations) determine the structure of the corresponding representation of $\mathcal{P}_n$ and $\mathcal{O}_n$. Our goal is to generalize the results obtained by Jones and Lawson (for $\mathcal{P}_n$) and by Bratteli and Jorgensen (for $\mathcal{O}_n$) on the periodic points in the case $n = 2$.

We determine the periodic points for arbitrary n when $d_1, \ldots, d_n$ is an arithmetic sequence (note that this is always the case if $n = 2$). It turns out that arithmetic sequences give in some sense the largest possible set of periodic points, and we provide a characterization of all other sequences yielding such a large set of periodic points. On the other extreme, we present infinite families of examples with a single periodic point. We also study the asymptotic behavior of the number of periodic points. We prove that the number of periodic points grows linearly if the parameters $d_1, \ldots, d_n$ tend to infinity in a proportional way, and we also give an asymptotically sharp upper estimate when one of the parameters tends to infinity with the others being fixed.

[Joint work with Miklós Hartmann (University of Szeged).]

Marc **Zeitoun** *University of Bordeaux*

Separation-like problems for regular languages

This talk focuses on some problems on classes of regular word languages. Such problems aim at capturing the expressiveness of logical or combinatorial formalisms. Since the work of Schützenberger, the most classical one is the "membership problem", which asks for an algorithm deciding whether a regular input language belongs to the class under investigation. Membership algorithms have been designed for many natural classes of languages, often originating from algebraic characterizations. However, some classes seem to require new conceptual ingredients.

Such ingredients introduced so far involve problems that are more demanding than membership, and may also be computationally harder. This may seem surprising at first, but can be explained by the fact that membership is not flexible enough as a framework. One such problem, defined by Henckell and Rhodes, consists in computing particular subsets of a semigroup, called "pointlike sets". The restriction of this problem to subsets of size 2 is already challenging, and has been given a nice formulation in terms of separation by Almeida (as well as the general problem). I will introduce these problems and try to convey the intuition of why they are indeed interesting.

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(3) Jorge Almeida, *University of Porto*
(4) Jorge André, *Universidade NOVA de Lisboa*
(5) João Araújo, *Universidade Aberta*
(6) Karl Auinger, *University of Vienna*
(7) Muhammed P. A. Azeef, *Indian Inst. of Sci. Edu. and Research*
(8) Bernard Oduoku Bainson, *Heriot Watt University*
(9) Wolfram Bentz, *University of Hull*
(10) Célia Borlido, *University of Porto*
(11) Tom Bourne, *University of St Andrews*
(12) Manuel B. Branco, *University of Évora*
(13) Mário Branco, *University of Lisbon*
(14) Tara Brough, *University of Lisbon*
(15) Alan Cain, *Universidade NOVA de Lisboa*
(16) Alonso Castillo-Ramirez, *Durham University*
(17) Eliana Castro, *University of Lisbon*
(18) Alessandra Cherubini, *Politecnico Milano*
(19) Maria Manuel Clementino, *University of Coimbra*
(20) Thomas Coleman, *University of East Anglia*
(21) Alfredo Costa, *University of Coimbra*
(22) José Carlos Costa, *University of Minho*
(23) Silke Czarnetzki, *University of Tuebingen*
(24) Tamás Dékány, *University of Szeged*
(25) Manuel Delgado, *University of Porto*
(26) Volker Diekert, *University of Stuttgart*
(27) Ana Paula Escada, *University of Coimbra*
(28) Ruy Exel, *Universidade Federal Santa Catarina*
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(30) Vítor Hugo Fernandes, *Universidade NOVA de Lisboa*
(31) John Fountain, *University of York*
(32) Maximilien Gadouleau, *Durham University*
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(34) Gracinda M. S. Gomes, *University of Lisbon*
(35) François Gonze, *University of Catholique de Louvain*
(36) Victoria Gould, *University of York*
(37) Maria João Gouveia, *University of Lisbon*
(38) Robert Gray, *University of East Anglia*
(39) Vladimir Gusev, *University Catholique de Louvain and Ural Federal University*
(40) Karsten Henckell, *New College of Florida*
(41) Tatiana Jajcayova, *Comenius University, Bratislava*
(42) Manuel M. Jesus, *Universidade NOVA de Lisboa*
(43) Mark Kambites, *University of Manchester*
(44) Kamilla Kátai-Urbán, *University of Szeged*
(45) Michael Kinyon, *University of Denver*
(46) Lukasz Kubat, *University of Warsaw*
(47) Ganna Kudryavtseva, *University of Ljubljana*
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(49) Lucinda Lima, *University of Porto*
(50) Markus Lohrey, *University of Siegen*
(51) Diane Lubbock, *University of East Anglia*
(52) António Malheiro, *Universidade NOVA de Lisboa*
(53) Stuart Margolis, *Bar Ilan University*
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(55) Nelson Martins-Ferreira, *Polytechnic Inst. Leiria*
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(57) Francesco Matucci, *Universidade Estadual de Campinas*
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